SPACE-TIME BLOCK CODES FROM CONSTITUENT INTERSECTANT ORTHOGONAL DESIGNS

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Abstract - In this paper, we propose space-time block codes based on constituent intersectant orthogonal designs. After channel model was formulated, we studied the space-time block codes from constituent intersectant orthogonal designs. At last we made simulations for the space-time block codes from complex orthogonal design and the space-time block codes from constituent intersectant orthogonal design to compare their performance. Compared with space-time block codes from complex orthogonal design when code rate is the same and transmission rate is fixed, space-time block codes from constituent intersectant orthogonal design with more transmit antennas can reduce bit error rate and symbol error rate, and these new codes have the same low decoding complexity as the space-time block codes from complex orthogonal designs.

Keywords - Space-Time Block Code, Constituent Intersectant Orthogonal Design, Complex Orthogonal Design, Decoding, Transmit Diversity.

INTRODUCTION

Nowadays, since the third generation mobile communication system requires much higher quality of speech than the current cellular mobile communication system and requires up to 2 *Mbils/s* high bit rate data service, more and more attention has been paid to transmit diversity of space-time coding. Space-time coding is a new encoding and signal processing technique in mobile communication system, which transmits and receives information using multiple transmit antennas and receive antennas. [1] Space-time processing can greatly improve information capacity and information rate. Space-time coding can provide more coding gain while not sacrificing transmitted bandwidth. Space-time codes are divided into space-time trellis codes and space-time block codes. [2] One of the design criteria of space-time block code is space-time block coding matrix satisfying orthogonal condition, i.e. orthogonal design. In this paper, we propose the space-time block codes based on constituent intersectant orthogonal design. These codes have the same low decoding complexity as the space-time block codes from complex orthogonal design. Rate I space-time block codes from constituent intersectant orthogonal design exist for two, three and four transmit antennas while space-time block codes from complex orthogonal design exist only for two transmit antennas. Rate 3/4 space-time block codes from constituent intersectant orthogonal design exist for five, six, seven and eight transmit antennas while space-time block codes from complex orthogonal design exist only for three and four transmit antennas. At last we make simulations for the space-time block codes from complex orthogonal design and the space-time block codes from constituent intersectant orthogonal design to compare their performance.

CHANNEL MODEL

We consider a wireless communication system which has n transmit antennas and m receive antennas. Let A be the $n \times m$ channel gain matrix. Thus the element a_{ij} *of* A is the (complex) gain factor for the path from the ith transmit antenna to the jth receive antenna. We assume that the received signal is corrupted by additive white Gaussian noise. Let W *be* the $p \times n$ matrix transmitted at time t. Then, the received $p \times m$ matrix R can be written as:

$$R = WA+N \tag{1}$$

where N *is* the $p \times m$ white noise matrix whose elements are independent identically distributed (i.i.d.) Gaussian random variables with mean zero and unit variance, and $tr(W^{+}W) = p$. The fading we consider here is frequency nonselective Rayleigh fading (or flat Rayleigh fading), in the case of which the elements of A are independent complex Gaussian random variables. [3]

Space-Time Block Codes Based on Constituent Intersectant Orthogonal Designs: A generalized complex linear processing orthogonal design of size n and rate k/p is a $p \times n$ matrix $G_c(x_1,...,x_k)$ in variables x_i, i = 1,…,k, which satisfies the following conditions: 1) the entries of $G_c(x_1,...,x_k)$ are complex linear combinations of $xi, i=1,...,k$ and their conjugales; 2) $G_c^{+}(x_1,...,x_k)G_c(x_1,...,x_k) = D$, where D is a diagonal matrix whose entries are a linear combination of $|x_i|^2$, $i= 1,...,k$ with all strictly positive real coefficients and Gc^{+} is the complex transpose conjugate of G_c. When $k=p=n$ we obtain complex linear processing orthogonal design. [4]

An example of a 2x2 complex orthogonal design is given by:

$$\begin{bmatrix} x_1 & x_2 \\ -x^*_2 & x^*_1 \end{bmatrix} \tag{2}$$

for n = 3 and 4, we construct rate 3/4 generalized complex linear processing orthogonal designs given by

$$\begin{pmatrix} x_1 & x_2 & \frac{x_3}{\sqrt{2}} \\ -x_2^* & x_1^* & \frac{x_3}{\sqrt{2}} \\ \frac{x_3^*}{\sqrt{2}} & \frac{x_3^*}{\sqrt{2}} & \frac{(-x_1-x_1^*+x_2-x_2^*)}{2} \\ \frac{x_3^*}{\sqrt{2}} & -\frac{x_3^*}{\sqrt{2}} & \frac{(x_2+x_2^*+x_1-x_1^*)}{2} \end{pmatrix} \quad (3)$$

for $n = 3$ and

$$\begin{pmatrix} x_1 & x_2 & \frac{x_3}{\sqrt{2}} & \frac{x_3}{\sqrt{2}} \\ -x_2^* & x_1^* & \frac{x_3}{\sqrt{2}} & -\frac{x_3}{\sqrt{2}} \\ \frac{x_3^*}{\sqrt{2}} & \frac{x_3^*}{\sqrt{2}} & \frac{(-x_1-x_1^*+x_2-x_2^*)}{2} & \frac{(-x_2-x_2^*+x_1-x_1^*)}{2} \\ \frac{x_3^*}{\sqrt{2}} & -\frac{x_3^*}{\sqrt{2}} & \frac{(x_2+x_2^*+x_1-x_1^*)}{2} & -\frac{(x_1+x_1^*+x_2-x_2^*)}{2} \end{pmatrix} \quad (4)$$

for n = 4. These codes are designed using the theory of orthogonal designs.

We now define space-time block codes based on constituent intersectant orthogonal designs. A constituent intersectant orthogonal design - of size n of rate k/p, in variables $x_i, i = 1,\ldots,k$ is a $p \times n$ matrix $W(x1,\ldots,xk)$ .such that:

$$W(x_1,\ldots,x_k) = \begin{bmatrix} Gc(\tilde{x}_1 \ldots,\tilde{x}_{k/2}) & Gc(\tilde{x}_{k/2+1},\ldots,\tilde{x}_k) \\ Gc(\tilde{x}_{k/2+1},\ldots,\tilde{x}_k) & Gc(\tilde{x}_1 \ldots,\tilde{x}_{k/2}) \end{bmatrix} \quad (5)$$

where Gc $(X_1,\ldots, X_{k/2})$ is a generalized complex linear processing orthogonal design of size n/2 of rate k/p, $x_i = \text{Re}\ \{x_i\} + j\ \text{Im}\ \{x_{(i+k/2)k}\}$ and where $(i + k/2)k$ denotes $(i + k/2)\ (mod\ k)$.

Examples of rate l, space-time block codes from constituent intersectant orthogonal design for $n = 2, 4$ are:

When $n = 2$, we have:

$$G_c(\tilde{x}_1,\ldots,\tilde{x}_{2/2}) = \tilde{x}_1 \text{ and } Gc(\tilde{x}_{2/2+1},\ldots,\tilde{x}_2) = \tilde{x}_2 \quad (6)$$

when $n = 4$ we can utilize (2):

$$Gc(\tilde{x}_1,\ldots,\tilde{x}_{4/2}) = Gc(\tilde{x}_1,\tilde{x}_2) = \begin{pmatrix} \tilde{x}_1 & \tilde{x}_2 \\ -x_2^* & x_1^* \end{pmatrix} \quad (7)$$

and

$$Gc(\tilde{X}_{k/2+1},\ldots,\tilde{X}_4)=Gc(\tilde{X}_3,\tilde{X}_4)=\begin{pmatrix}\tilde{X}_3 & \tilde{X}_4\\ -X_4^* & X_3^*\end{pmatrix} \tag{8}$$

From the above formula we can get the following:

$$W^2_{(X1,X2)}=\begin{pmatrix}\tilde{X}_1 & \tilde{X}_2\\ \tilde{X}_2 & \tilde{X}_1\end{pmatrix} \tag{9}$$

$$W^4_{(X1,\ldots,X4)}=\begin{pmatrix}\tilde{X}_1 & \tilde{X}_2 & \tilde{X}_3 & \tilde{X}_4\\ -\tilde{X}_2^* & \tilde{X}_1^* & -\tilde{X}_4^* & \tilde{X}_3^*\\ \tilde{X}_3 & \tilde{X}_4 & \tilde{X}_1 & \tilde{X}_2\\ -\tilde{X}_3^* & \tilde{X}_3^* & -\tilde{X}_2^* & \tilde{X}_1^*\end{pmatrix} \tag{10}$$

A rate I constituent intersectant orthogonal design matrix for $n = 3$ can be obtained from n=4, i.e. by deleting one of the columns of constituent intersectant orthogonal design matrix (10):

$$W^3_{(X1,\ldots X4)}=\begin{pmatrix}\tilde{X}_1 & \tilde{X}_3 & \tilde{X}_4\\ -\tilde{X}_2^* & -\tilde{X}_4^* & \tilde{X}_3^*\\ \tilde{X}_3 & \tilde{X}_1 & \tilde{X}_2\\ -\tilde{X}_4^* & -\tilde{X}_2^* & \tilde{X}_1^*\end{pmatrix},\ W^3_{(X1,\ldots,X4)}=\begin{pmatrix}\tilde{X}_1 & \tilde{X}_2 & \tilde{X}_4\\ -\tilde{X}_2^* & \tilde{X}_1^* & \tilde{X}_3^*\\ \tilde{X}_3 & \tilde{X}_4 & \tilde{X}_2\\ -\tilde{X}_4^* & \tilde{X}_3^* & \tilde{X}_1^*\end{pmatrix},\ W^3_{(X1,\ldots,X4)}=\begin{pmatrix}\tilde{X}_2 & \tilde{X}_3 & \tilde{X}_4\\ \tilde{X}_1^* & -\tilde{X}_4 & \tilde{X}_3^*\\ \tilde{X}_4 & \tilde{X}_1 & \tilde{X}_2\\ \tilde{X}_1^* & -\tilde{X}_2^* & \tilde{X}_1^*\end{pmatrix},$$

$$W^3_{(X1,\ldots,X4)}=\begin{pmatrix}\tilde{X}_1 & \tilde{X}_2 & \tilde{X}_3\\ -\tilde{X}_2^* & \tilde{X}_1^* & -\tilde{X}_4^*\\ \tilde{X}_3 & \tilde{X}_4 & \tilde{X}_1\\ -\tilde{X}_4^* & \tilde{X}_3^* & -\tilde{X}_2^*\end{pmatrix}$$

We can thus get the following theorem.

Theorem: Space-time block codes based on constituent intersectant orthogonal design of rate 1 of size n exists if and only if $n = 2$, 3 or 4. Similarly, space-time block codes based on constituent intersectant orthogonal design of rate 3/4 of size n exists if and only if n = 5, 6, 7 or 8.

Proof: Suppose a rate 1, Space-Time block codes based on constituent mier-sectant orthogonal design of size n exists when $n = 6$. Let $k = 2$, then *we* can get a 2×2 constituent intersectant orthogonal design matrix from equation (5) which is the definition of constituent intersectant orthogonal design. This is in contradiction with the fact that we should get a p×n (i.e. 2×6) constituent intersectant orthogonal design matrix. Let $k = 4$, then we can get a 4×4 constituent intersectant orthogonal design matrix from (5) and (2), which is in contradiction with the fact that we should get a p×n (i.e. 4×6) constituent intersectant orthogonal design matrix Let $k=6$, since complex orthogonal design exists if and only if n = 2, while $G_c(\tilde{X}_{k/2+1},\dots,\tilde{X}_6)$ and $G_c(\tilde{X}_1,\dots,\tilde{X}_{6/2})$ both are 3×3 matrix when $k = 6$, this kind of matrices don't exist at all. Thus constituent intersectant orthogonal design matrix doesn't exist when $k = 6$. We can also prove that rate 1, constituent intersectant orthogonal designs of size n does not exist when $n > 4$.

If G_c is a generalized complex linear processing orthogonal design of size 4, rate 3/4, then we have rate 3/4 constituent intersectant orthogonal design codes for $n = 5, 6, 7, 8$. We can get $k = 6, p = 8$ when n = 6, then we can get a 8 × 6 matrix by utilizing (3) as follows:

$$Gc(\tilde{X}_1,\dots,\tilde{X}_{6/2}) = Gc(\tilde{X}_1,\tilde{X}_2,\tilde{X}_3) = \begin{pmatrix} \tilde{X}_1 & x_2 & \frac{\tilde{X}_3}{\sqrt{2}} \\ -\tilde{X}_2^* & \tilde{X}_1^* & \frac{\tilde{X}_3}{\sqrt{2}} \\ \frac{\tilde{X}_3^*}{\sqrt{2}} & \frac{\tilde{X}_3^*}{\sqrt{2}} & \frac{(-\tilde{X}_1 - \tilde{X}_1^* + \tilde{X}_2 - \tilde{X}_2^*)}{2} \\ \frac{\tilde{X}_3^*}{\sqrt{2}} & -\frac{\tilde{X}_3^*}{\sqrt{2}} & \frac{(\tilde{X}_2 + \tilde{X}_2^* + \tilde{X}_1 - \tilde{X}_1^*)}{2} \end{pmatrix} \tag{11}$$

and

$$Gc(\tilde{X}_{k/2+1},\dots,\tilde{X}_k) = Gc(\tilde{X}_4,\tilde{X}_5,\tilde{X}_6) = \begin{pmatrix} \tilde{X}_4 & X_5 & \frac{\tilde{X}_6}{\sqrt{2}} \\ -\tilde{X}_5^* & \tilde{X}_4^* & \frac{\tilde{X}_6}{\sqrt{2}} \\ \frac{\tilde{X}_6^*}{\sqrt{2}} & \frac{\tilde{X}_6^*}{\sqrt{2}} & \frac{(-\tilde{X}_4 - \tilde{X}_4^* + \tilde{X}_5 - \tilde{X}_5^*)}{2} \\ \frac{\tilde{X}_6^*}{\sqrt{2}} & -\frac{\tilde{X}_6^*}{\sqrt{2}} & \frac{(\tilde{X}_5 + \tilde{X}_5^* + \tilde{X}_4 - \tilde{X}_4^*)}{2} \end{pmatrix} \tag{12}$$

we can then get $W^6_{(X1,\dots,X6)}$ as follows:

$$W^{6}(X1,\dots,X6)=\begin{pmatrix}
\tilde{x}_1 & x_2 & \frac{\tilde{x}_3}{\sqrt{2}} & \tilde{x}_4 & x_5 & \frac{\tilde{x}_6}{\sqrt{2}} \\
-\tilde{x}_2 & \tilde{x}_1^* & \frac{\tilde{x}_3}{\sqrt{2}} & -\tilde{x}_5^* & \tilde{x}_4^* & \frac{\tilde{x}_6}{\sqrt{2}} \\
\frac{\tilde{x}_3^*}{\sqrt{2}} & \frac{\tilde{x}_3^*}{\sqrt{2}} & \frac{(-\tilde{x}_1-\tilde{x}_1^*+\tilde{x}_2-\tilde{x}_2^*)}{2} & \frac{\tilde{x}_6^*}{\sqrt{2}} & \frac{\tilde{x}_6^*}{\sqrt{2}} & \frac{(-\tilde{x}_4-\tilde{x}_4^*+\tilde{x}_5-\tilde{x}_5^*)}{2} \\
\frac{\tilde{x}_3^*}{\sqrt{2}} & -\frac{\tilde{x}_3^*}{\sqrt{2}} & \frac{(\tilde{x}_2+\tilde{x}_2^*+\tilde{x}_1-\tilde{x}_1^*)}{2} & \frac{\tilde{x}_6^*}{\sqrt{2}} & -\frac{\tilde{x}_6^*}{\sqrt{2}} & \frac{(\tilde{x}_5+\tilde{x}_5^*+\tilde{x}_4-\tilde{x}_4^*)}{2} \\
\tilde{x}_4 & \tilde{x}_5 & \frac{\tilde{x}_6}{\sqrt{2}} & \tilde{x}_1 & \tilde{x}_2 & \frac{\tilde{x}_3}{\sqrt{2}} \\
-\tilde{x}_5^* & \tilde{x}_4 & \frac{\tilde{x}_6}{\sqrt{2}} & -\tilde{x}_2^* & \tilde{x}_1^* & \frac{\tilde{x}_3}{\sqrt{2}} \\
\frac{\tilde{x}_6^*}{\sqrt{2}} & \frac{\tilde{x}_6^*}{\sqrt{2}} & \frac{(-\tilde{x}_4-\tilde{x}_4^*+\tilde{x}_5-\tilde{x}_5^*)}{2} & \frac{\tilde{x}_3^*}{\sqrt{2}} & \frac{\tilde{x}_3^*}{\sqrt{2}} & \frac{(-\tilde{x}_1-\tilde{x}_1^*+\tilde{x}_2-\tilde{x}_2^*)}{2} \\
\frac{\tilde{x}_6^*}{\sqrt{2}} & -\frac{\tilde{x}_6^*}{\sqrt{2}} & \frac{(\tilde{x}_5+\tilde{x}_5^*+\tilde{x}_4-\tilde{x}_4^*)}{2} & \frac{\tilde{x}_3^*}{\sqrt{2}} & -\frac{\tilde{x}_3^*}{\sqrt{2}} & \frac{(\tilde{x}_2+\tilde{x}_2^*+\tilde{x}_1-\tilde{x}_1^*)}{2}
\end{pmatrix} \quad (13)$$

A rate 3/4 constitutent intersectant orthogonal design, matrix for n = 5 can be obtained from n = 6, i.e. by deleting one of the columns of constituent intersectant orthogonal design matrix (13). In other words:

$W^5(X_1, \dots, X_6) =:$

$$\begin{pmatrix}
\tilde{X}_1 & \tilde{X}_2 & \tilde{X}_4 & \tilde{X}_5 & \frac{\tilde{X}_6}{\sqrt{2}} \\
-\tilde{X}_2^* & \tilde{X}_1^* & -\tilde{X}_5^* & \tilde{X}_4^* & \frac{\tilde{X}_6}{\sqrt{2}} \\
\frac{\tilde{X}_3^*}{\sqrt{2}} & \frac{\tilde{X}_3^*}{\sqrt{2}} & \frac{\tilde{X}_6^*}{\sqrt{2}} & \frac{\tilde{X}_6^*}{\sqrt{2}} & \frac{(-\tilde{X}_4-\tilde{X}_4^*+\tilde{X}_5-\tilde{X}_5^*)}{2} \\
\frac{\tilde{X}_3^*}{\sqrt{2}} & -\frac{\tilde{X}_3^*}{\sqrt{2}} & \frac{\tilde{X}_6^*}{\sqrt{2}} & -\frac{\tilde{X}_6^*}{\sqrt{2}} & \frac{(\tilde{X}_5+\tilde{X}_5^*+\tilde{X}_4-\tilde{X}_4^*)}{2} \\
\tilde{X}_4 & \tilde{X}_5 & \tilde{X}_1 & \tilde{X}_2 & \frac{\tilde{X}_3}{\sqrt{2}} \\
-\tilde{X}_5^* & \tilde{X}_4^* & -\tilde{X}_2^* & \tilde{X}_1^* & \frac{\tilde{X}_3}{\sqrt{2}} \\
\frac{\tilde{X}_6^*}{\sqrt{2}} & \frac{\tilde{X}_6^*}{\sqrt{2}} & \frac{\tilde{X}_3^*}{\sqrt{2}} & \frac{\tilde{X}_3^*}{\sqrt{2}} & \frac{(-\tilde{X}_1-\tilde{X}_1^*+\tilde{X}_2-\tilde{X}_2^*)}{2} \\
\frac{\tilde{X}_6^*}{\sqrt{2}} & -\frac{\tilde{X}_6^*}{\sqrt{2}} & \frac{\tilde{X}_3^*}{\sqrt{2}} & -\frac{\tilde{X}_3^*}{\sqrt{2}} & \frac{(\tilde{X}_2+\tilde{X}_2^*+\tilde{X}_1-\tilde{X}_1^*)}{2}
\end{pmatrix}$$

or

$$\begin{pmatrix}
\tilde{X}_1 & \frac{\tilde{X}_3}{\sqrt{2}} & \tilde{X}_4 & \tilde{X}_5 & \frac{\tilde{X}_6}{\sqrt{2}} \\
-\dot{\tilde{X}}_2 & \frac{\tilde{X}_3}{\sqrt{2}} & -\dot{\tilde{X}}_5 & \dot{\tilde{X}}_4 & \frac{\tilde{X}_6}{\sqrt{2}} \\
\frac{\dot{\tilde{X}}_3}{\sqrt{2}} & \frac{(-\dot{\tilde{X}}_1-\dot{\tilde{X}}_1+\tilde{X}_2-\dot{\tilde{X}}_2)}{2} & \frac{\dot{\tilde{X}}_6}{\sqrt{2}} & \frac{\dot{\tilde{X}}_6}{\sqrt{2}} & \frac{(-\tilde{X}_4-\dot{\tilde{X}}_4+\tilde{X}_5-\dot{\tilde{X}}_5)}{2} \\
\frac{\dot{\tilde{X}}_3}{\sqrt{2}} & \frac{(\tilde{X}_2+\dot{\tilde{X}}_2+\tilde{X}_1-\dot{\tilde{X}}_1)}{2} & \frac{\dot{\tilde{X}}_6}{\sqrt{2}} & -\frac{\dot{\tilde{X}}_6}{\sqrt{2}} & \frac{(\tilde{X}_5+\dot{\tilde{X}}_5+\tilde{X}_4-\dot{\tilde{X}}_4)}{2} \\
\tilde{X}_4 & \frac{\tilde{X}_6}{\sqrt{2}} & \tilde{X}_1 & \tilde{X}_2 & \frac{\tilde{X}_3}{\sqrt{2}} \\
-\dot{\tilde{X}}_5 & \frac{\tilde{X}_6}{\sqrt{2}} & -\dot{\tilde{X}}_2 & \dot{\tilde{X}}_1 & \frac{\tilde{X}_3}{\sqrt{2}} \\
\frac{\dot{\tilde{X}}_6}{\sqrt{2}} & \frac{(-\tilde{X}_4-\dot{\tilde{X}}_4+\tilde{X}_5-\dot{\tilde{X}}_5)}{2} & \frac{\dot{\tilde{X}}_3}{\sqrt{2}} & \frac{\dot{\tilde{X}}_3}{\sqrt{2}} & \frac{(-\tilde{X}_1-\dot{\tilde{X}}_1+\tilde{X}_2-\dot{\tilde{X}}_2)}{2} \\
\frac{\dot{\tilde{X}}_6}{\sqrt{2}} & \frac{(\tilde{X}_5+\dot{\tilde{X}}_5+\tilde{X}_4-\dot{\tilde{X}}_4)}{2} & \frac{\dot{\tilde{X}}_3}{\sqrt{2}} & -\frac{\tilde{X}_3}{\sqrt{2}} & \frac{(\tilde{X}_2+\dot{\tilde{X}}_2+\tilde{X}_1-\dot{\tilde{X}}_1)}{2}
\end{pmatrix}$$

or

$$\begin{pmatrix}
\tilde{X}_2 & \frac{\tilde{X}_3}{\sqrt{2}} & \tilde{X}_4 & \tilde{X}_5 & \frac{\tilde{X}_6}{\sqrt{2}} \\
\dot{\tilde{X}}_1 & \frac{\tilde{X}_3}{\sqrt{2}} & -\dot{\tilde{X}}_5 & \dot{\tilde{X}}_4 & \frac{\tilde{X}_6}{\sqrt{2}} \\
\frac{\dot{\tilde{X}}_3}{\sqrt{2}} & \frac{(-\tilde{X}_1-\dot{\tilde{X}}_1+\tilde{X}_2-\dot{\tilde{X}}_2)}{2} & \frac{\dot{\tilde{X}}_6}{\sqrt{2}} & \frac{\dot{\tilde{X}}_6}{\sqrt{2}} & \frac{(-\tilde{X}_4-\dot{\tilde{X}}_4+\tilde{X}_5-\dot{\tilde{X}}_5)}{2} \\
-\frac{\dot{\tilde{X}}_3}{\sqrt{2}} & \frac{(\tilde{X}_2+\dot{\tilde{X}}_2+\tilde{X}_1-\dot{\tilde{X}}_1)}{2} & \frac{\dot{\tilde{X}}_6}{\sqrt{2}} & -\frac{\dot{\tilde{X}}_6}{\sqrt{2}} & \frac{(\tilde{X}_5+\dot{\tilde{X}}_5+\tilde{X}_4-\dot{\tilde{X}}_4)}{2} \\
\tilde{X}_5 & \frac{\tilde{X}_6}{\sqrt{2}} & \tilde{X}_1 & \tilde{X}_2 & \frac{\tilde{X}_3}{\sqrt{2}} \\
\dot{\tilde{X}}_4 & \frac{\tilde{X}_6}{\sqrt{2}} & -\dot{\tilde{X}}_2 & \dot{\tilde{X}}_1 & \frac{\tilde{X}_3}{\sqrt{2}} \\
\frac{\dot{\tilde{X}}_6}{\sqrt{2}} & \frac{(-\tilde{X}_4-\dot{\tilde{X}}_4+\tilde{X}_5-\dot{\tilde{X}}_5)}{2} & \frac{\dot{\tilde{X}}_3}{\sqrt{2}} & \frac{\dot{\tilde{X}}_3}{\sqrt{2}} & \frac{(-\tilde{X}_1-\dot{\tilde{X}}_1+\tilde{X}_2-\dot{\tilde{X}}_2)}{2} \\
-\frac{\dot{\tilde{X}}_6}{\sqrt{2}} & \frac{(\tilde{X}_5+\dot{\tilde{X}}_5+\tilde{X}_4-\dot{\tilde{X}}_4)}{2} & \frac{\dot{\tilde{X}}_3}{\sqrt{2}} & -\frac{\tilde{X}_3}{\sqrt{2}} & \frac{(\tilde{X}_2+\dot{\tilde{X}}_2+\tilde{X}_1-\dot{\tilde{X}}_1)}{2}
\end{pmatrix}$$

or

$$\begin{pmatrix}
\tilde{X}_1 & \tilde{X}_2 & \frac{\tilde{X}_3}{\sqrt{2}} & \tilde{X}_4 & \tilde{X}_5 \\
-\tilde{X}_2^* & \tilde{X}_1^* & \frac{\tilde{X}_3}{\sqrt{2}} & -\tilde{X}_5^* & \tilde{X}_4^* \\
\frac{\tilde{X}_3^*}{\sqrt{2}} & \frac{\tilde{X}_3^*}{\sqrt{2}} & \frac{(-\tilde{X}_1 - \tilde{X}_1^* + \tilde{X}_2 - \tilde{X}_2^*)}{2} & \frac{\tilde{X}_6^*}{\sqrt{2}} & \frac{\tilde{X}_6^*}{\sqrt{2}} \\
\frac{\tilde{X}_3^*}{\sqrt{2}} & -\frac{\tilde{X}_3^*}{\sqrt{2}} & \frac{(\tilde{X}_2 + \tilde{X}_2^* + \tilde{X}_1 - \tilde{X}_1^*)}{2} & \frac{\tilde{X}_6^*}{\sqrt{2}} & -\frac{\tilde{X}_6^*}{\sqrt{2}} \\
\tilde{X}_4 & \tilde{X}_5 & \frac{\tilde{X}_6}{\sqrt{2}} & \tilde{X}_1 & \tilde{X}_2 \\
-\tilde{X}_5^* & \tilde{X}_4^* & \frac{\tilde{X}_6}{\sqrt{2}} & -\tilde{X}_2^* & \tilde{X}_1^* \\
\frac{\tilde{X}_6^*}{\sqrt{2}} & \frac{\tilde{X}_6^*}{\sqrt{2}} & \frac{(-\tilde{X}_4 - \tilde{X}_4^* + \tilde{X}_5 - \tilde{X}_5^*)}{2} & \frac{\tilde{X}_3^*}{\sqrt{2}} & \frac{\tilde{X}_3^*}{\sqrt{2}} \\
\frac{\tilde{X}_6^*}{\sqrt{2}} & -\frac{\tilde{X}_6^*}{\sqrt{2}} & \frac{(\tilde{X}_5 + \tilde{X}_5^* + \tilde{X}_4 - \tilde{X}_4^*)}{2} & \frac{\tilde{X}_3^*}{\sqrt{2}} & 0
\end{pmatrix}$$

$$\begin{pmatrix}
\tilde{X}_1 & \tilde{X}_2 & \frac{\tilde{X}_3}{\sqrt{2}} & \tilde{X}_4 & \frac{\tilde{X}_6}{\sqrt{2}} \\
-\tilde{X}_2^* & \tilde{X}_1^* & \frac{\tilde{X}_3}{\sqrt{2}} & -\tilde{X}_5 & \frac{\tilde{X}_6}{\sqrt{2}} \\
\frac{\tilde{X}_3^*}{\sqrt{2}} & \frac{\tilde{X}_3^*}{\sqrt{2}} & \frac{(-\tilde{X}_1 - \tilde{X}_1^* + \tilde{X}_2 - \tilde{X}_2^*)}{2} & \frac{\tilde{X}_6^*}{\sqrt{2}} & \frac{(-\tilde{X}_4 - \tilde{X}_4^* + \tilde{X}_5 - \tilde{X}_5^*)}{2} \\
\frac{\tilde{X}_3^*}{\sqrt{2}} & -\frac{\tilde{X}_3^*}{\sqrt{2}} & \frac{(\tilde{X}_2 + \tilde{X}_2^* + \tilde{X}_1 - \tilde{X}_1^*)}{2} & \frac{\tilde{X}_6^*}{\sqrt{2}} & \frac{(\tilde{X}_5 + \tilde{X}_5^* + \tilde{X}_4 - \tilde{X}_4^*)}{2} \\
\tilde{X}_4 & \tilde{X}_5 & \frac{\tilde{X}_6}{\sqrt{2}} & \tilde{X}_1 & \frac{\tilde{X}_3}{\sqrt{2}} \\
-\tilde{X}_5^* & \tilde{X}_4^* & \frac{\tilde{X}_6}{\sqrt{2}} & -\tilde{X}_2^* & \frac{\tilde{X}_3}{\sqrt{2}} \\
\frac{\tilde{X}_6^*}{\sqrt{2}} & \frac{\tilde{X}_6^*}{\sqrt{2}} & \frac{(-\tilde{X}_4 - \tilde{X}_4^* + \tilde{X}_5 - \tilde{X}_5^*)}{2} & \frac{\tilde{X}_3^*}{\sqrt{2}} & \frac{(-\tilde{X}_1 - \tilde{X}_1^* + \tilde{X}_2 - \tilde{X}_2^*)}{2} \\
\frac{\tilde{X}_6^*}{\sqrt{2}} & -\frac{\tilde{X}_6^*}{\sqrt{2}} & \frac{(\tilde{X}_5 + \tilde{X}_5^* + \tilde{X}_4 - \tilde{X}_4^*)}{2} & \frac{\tilde{X}_3^*}{\sqrt{2}} & \frac{(\tilde{X}_2 + \tilde{X}_2^* + \tilde{X}_1 - \tilde{X}_1^*)}{2}
\end{pmatrix}$$

$$\begin{pmatrix}
\tilde{x}_1 & \tilde{x}_2 & \frac{\tilde{x}_3}{\sqrt{2}} & \tilde{x}_5 & \frac{\tilde{x}_6}{\sqrt{2}} \\
-\dot{\tilde{x}}_2 & \dot{\tilde{x}}_1 & \frac{\tilde{x}_3}{\sqrt{2}} & \tilde{x}_4 & \frac{\tilde{x}_6}{\sqrt{2}} \\
\frac{\dot{\tilde{x}}_3}{\sqrt{2}} & \frac{\dot{\tilde{x}}_3}{\sqrt{2}} & \frac{(-\tilde{x}_1-\dot{\tilde{x}}_1+\tilde{x}_2-\dot{\tilde{x}}_2)}{2} & \frac{\dot{\tilde{x}}_6}{\sqrt{2}} & \frac{(-\tilde{x}_4-\dot{\tilde{x}}_4+\tilde{x}_5-\dot{\tilde{x}}_5)}{2} \\
\frac{\dot{\tilde{x}}_3}{\sqrt{2}} & -\frac{\dot{\tilde{x}}_3}{\sqrt{2}} & \frac{(\tilde{x}_2+\dot{\tilde{x}}_2+\tilde{x}_1-\dot{\tilde{x}}_1)}{2} & -\frac{\dot{\tilde{x}}_6}{\sqrt{2}} & \frac{(\tilde{x}_5+\dot{\tilde{x}}_5+\tilde{x}_4-\dot{\tilde{x}}_4)}{2} \\
\tilde{x}_4 & \tilde{x}_5 & \frac{\tilde{x}_6}{\sqrt{2}} & \tilde{x}_2 & \frac{\tilde{x}_3}{\sqrt{2}} \\
-\dot{\tilde{x}}_5 & \dot{\tilde{x}}_4 & \frac{\tilde{x}_6}{\sqrt{2}} & -\dot{\tilde{x}}_1 & \frac{\tilde{x}_3}{\sqrt{2}} \\
\frac{\dot{\tilde{x}}_6}{\sqrt{2}} & \frac{\dot{\tilde{x}}_6}{\sqrt{2}} & \frac{(-\tilde{x}_4-\dot{\tilde{x}}_4+\tilde{x}_5-\dot{\tilde{x}}_5)}{2} & \frac{\dot{\tilde{x}}_3}{\sqrt{2}} & \frac{(-\tilde{x}_1-\dot{\tilde{x}}_1+\tilde{x}_2-\dot{\tilde{x}}_2)}{2} \\
\frac{\dot{\tilde{x}}_6}{\sqrt{2}} & -\frac{\dot{\tilde{x}}_6}{\sqrt{2}} & \frac{(\tilde{x}_5+\dot{\tilde{x}}_5+\tilde{x}_4-\dot{\tilde{x}}_4)}{2} & -\frac{\dot{\tilde{x}}_3}{\sqrt{2}} & \frac{(\tilde{x}_2+\dot{\tilde{x}}_2+\tilde{x}_1-\dot{\tilde{x}}_1)}{2}
\end{pmatrix}$$

When $n = 8$, we can get $k = 6$, $p = 8$. Then we get a 8×8 matrix by utilizing (4) as follows:

$$Gc(\tilde{x}_1,\dots,\tilde{x}_{6/2}) = Gc(\tilde{x}_1,\tilde{x}_2,\tilde{x}_3) = \begin{pmatrix}
\tilde{x}_1 & \tilde{x}_2 & \frac{\tilde{x}_3}{\sqrt{2}} & \frac{\tilde{x}_3}{\sqrt{2}} \\
-\dot{\tilde{x}}_2 & \dot{\tilde{x}}_1 & \frac{\tilde{x}_3}{\sqrt{2}} & -\frac{\tilde{x}_3}{\sqrt{2}} \\
\frac{\dot{\tilde{x}}_3}{\sqrt{2}} & \frac{\dot{\tilde{x}}_3}{\sqrt{2}} & \frac{(-\tilde{x}_1-\dot{\tilde{x}}_1+\tilde{x}_2-\dot{\tilde{x}}_2)}{2} & \frac{(-\tilde{x}_2-\dot{\tilde{x}}_2+\tilde{x}_1-\dot{\tilde{x}}_1)}{2} \\
\frac{\dot{\tilde{x}}_3}{\sqrt{2}} & -\frac{\dot{\tilde{x}}_3}{\sqrt{2}} & \frac{(\tilde{x}_2+\dot{\tilde{x}}_2+\tilde{x}_1-\dot{\tilde{x}}_1)}{2} & -\frac{(\tilde{x}_1+\dot{\tilde{x}}_1+\tilde{x}_2-\dot{\tilde{x}}_2)}{2}
\end{pmatrix} \qquad (14)$$

and

$$Gc(\tilde{x}_{6/2+1},\dots,\tilde{x}_6) = Gc(\tilde{x}_4,\tilde{x}_5,\tilde{x}_6) = \begin{pmatrix}
\tilde{x}_4 & \tilde{x}_5 & \frac{\tilde{x}_6}{\sqrt{2}} & \frac{\tilde{x}_6}{\sqrt{2}} \\
-\dot{\tilde{x}}_5 & \dot{\tilde{x}}_4 & \frac{\tilde{x}_6}{\sqrt{2}} & -\frac{\tilde{x}_6}{\sqrt{2}} \\
\frac{\dot{\tilde{x}}_6}{\sqrt{2}} & \frac{\dot{\tilde{x}}_6}{\sqrt{2}} & \frac{(-\tilde{x}_4-\dot{\tilde{x}}_4+\tilde{x}_5-\dot{\tilde{x}}_5)}{2} & \frac{(-\tilde{x}_5-\dot{\tilde{x}}_5+\tilde{x}_4-\dot{\tilde{x}}_4)}{2} \\
\frac{\dot{\tilde{x}}_6}{\sqrt{2}} & -\frac{\dot{\tilde{x}}_6}{\sqrt{2}} & \frac{(\tilde{x}_5+\dot{\tilde{x}}_5+\tilde{x}_4-\dot{\tilde{x}}_4)}{2} & -\frac{(\tilde{x}_4+\dot{\tilde{x}}_4+\tilde{x}_5-\dot{\tilde{x}}_5)}{2}
\end{pmatrix} \qquad (15)$$

We can then get $W^8_{(x1,\dots,x6)}$ as follows:

$$W^{8}_{(x_1,\dots,x_6)}=\begin{pmatrix}
x_1 & \tilde{x}_2 & \frac{\tilde{x}_3}{\sqrt{2}} & \frac{\tilde{x}_3}{\sqrt{2}} & \tilde{x}_1 & \tilde{x}_2 & \frac{\tilde{x}_3}{\sqrt{2}} & \frac{x_3}{\sqrt{2}} \\
-x_2^* & \tilde{x}_1^* & \frac{\tilde{x}_3}{\sqrt{2}} & -\frac{\tilde{x}_3}{\sqrt{2}} & -\tilde{x}_2^* & \tilde{x}_1^* & \frac{\tilde{x}_3}{\sqrt{2}} & -\frac{x_3}{\sqrt{2}} \\
\frac{\tilde{x}_3^*}{\sqrt{2}} & \frac{\tilde{x}_3^*}{\sqrt{2}} & \frac{(-\tilde{x}_1-\tilde{x}_1^*+\tilde{x}_2-\tilde{x}_2^*)}{2} & \frac{(-\tilde{x}_2-\tilde{x}_2^*+\tilde{x}_1-\tilde{x}_1^*)}{2} & \frac{\tilde{x}_3^*}{\sqrt{2}} & \frac{\tilde{x}_3^*}{\sqrt{2}} & \frac{(-\tilde{x}_1-x_1^*+\tilde{x}_2-\tilde{x}_2^*)}{2} & \frac{(-\tilde{x}_1-\tilde{x}_1^*+\tilde{x}_2-x_2^*)}{2} \\
\frac{x_3^*}{\sqrt{2}} & -\frac{\tilde{x}_3^*}{\sqrt{2}} & \frac{(\tilde{x}_2+\tilde{x}_2^*+\tilde{x}_1-\tilde{x}_1^*)}{2} & -\frac{(\tilde{x}_1+\tilde{x}_1^*+\tilde{x}_2-\tilde{x}_2^*)}{2} & \frac{\tilde{x}_3^*}{\sqrt{2}} & \frac{\tilde{x}_3^*}{\sqrt{2}} & \frac{(\tilde{x}_2+\tilde{x}_2^*+x_1-\tilde{x}_1^*)}{2} & -\frac{(\tilde{x}_1+\tilde{x}_1^*+\tilde{x}_2-x_2^*)}{2} \\
x_4 & x_5 & \frac{\tilde{x}_6}{\sqrt{2}} & \frac{\tilde{x}_6}{\sqrt{2}} & \tilde{x}_1 & \tilde{x}_2 & \frac{\tilde{x}_3}{\sqrt{2}} & \frac{\tilde{x}_3}{\sqrt{2}} \\
\tilde{x}_5^* & \tilde{x}_4^* & \frac{x_6}{\sqrt{2}} & -\frac{\tilde{x}_6}{\sqrt{2}} & -\tilde{x}_2^* & \tilde{x}_1^* & \frac{\tilde{x}_3}{\sqrt{2}} & -\frac{x_3}{\sqrt{2}} \\
\frac{x_6^*}{\sqrt{2}} & \frac{\tilde{x}_6^*}{\sqrt{2}} & \frac{(-\tilde{x}_4-\tilde{x}_4^*+\tilde{x}_5-\tilde{x}_5^*)}{2} & \frac{(-\tilde{x}_5-\tilde{x}_5^*+\tilde{x}_4-\tilde{x}_4^*)}{2} & \frac{\tilde{x}_1^*}{\sqrt{2}} & \frac{\tilde{x}_3^*}{\sqrt{2}} & \frac{(-\tilde{x}_1-\tilde{x}_1^*+x_2-\tilde{x}_2^*)}{2} & \frac{(-\tilde{x}_2-x_2^*+\tilde{x}_1-\tilde{x}_1^*)}{2} \\
\frac{x_3^*}{\sqrt{2}} & -\frac{\tilde{x}_3^*}{\sqrt{2}} & \frac{(\tilde{x}_5+\tilde{x}_5^*+x_4-\tilde{x}_4^*)}{2} & -\frac{(\tilde{x}_4+\tilde{x}_4^*+\tilde{x}_5-\tilde{x}_5^*)}{2} & \frac{\tilde{x}_3^*}{\sqrt{2}} & -\frac{\tilde{x}_3^*}{\sqrt{2}} & \frac{(\tilde{x}_2+\tilde{x}_2^*+\tilde{x}_1^*-x_1^*)}{2} & -\frac{(\tilde{x}_1+\tilde{x}_1^*+\tilde{x}_2-x_2^*)}{2}
\end{pmatrix} \quad (16)$$

A rate 3/4 constituent intersectant orthogonal design matrix for n = 7 can be obtained from n = 8, i.e. by deleting one of the columns of constituent intersectant orthogonal design matrix (16).

SIMULATIONS

We make simulations for rate 1 space-time block codes from complex orthogonal design of n = 2 and rate 1 space-time block codes from constituent intersectant orthogonal design of n = 3, 4, and make simulations for rate 3/4 space-time block codes from complex orthogonal design of n = 3, 4 and rate 3/4 space-time block codes from constituent intersectant orthogonal design of n = 6, 8. Transmission rate is 1bits/s/Hz, $m = 1$. Simulation results are shown in Figure 1 and Figure 2. Simulation results show that rate 1 space-time block codes from constituent intersectant orthogonal design exist for two, three and four transmit antennas while space-time block codes from complex orthogonal design exists only for two transmit antennas. Also, rate 3/4 space-time block codes from constituent intersectant orthogonal design exist for five, six, seven and eight transmit antennas while space-time block codes from complex orthogonal design exist only for three and four transmit antennas. Compared with space-time block codes from complex orthogonal design when code rate is the same and transmission rate is fixed, space-time block codes from constituent intersectant orthogonal design with more transmit antennas can reduce bit error rate and symbol error rate.

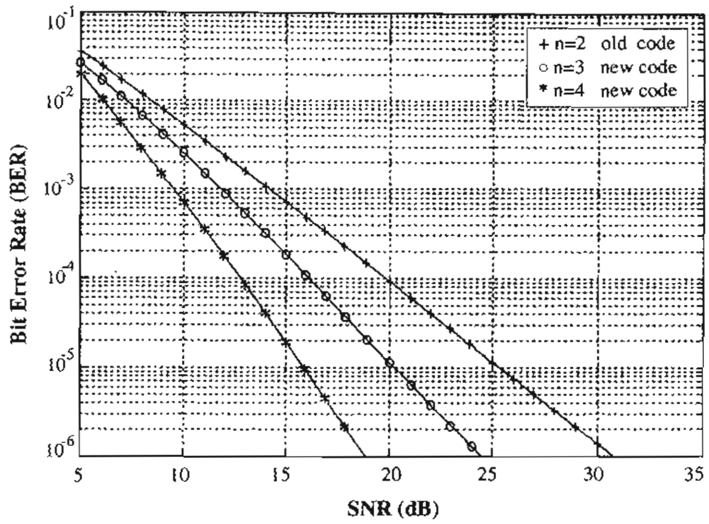

Figure 1: Rate l space-time block codes from complex orthogonal design (old codes) of $n = 2$ and space-time block codes from constituent intersectant orthogonal design (new codes) of $n = 3,4$ (m = l)

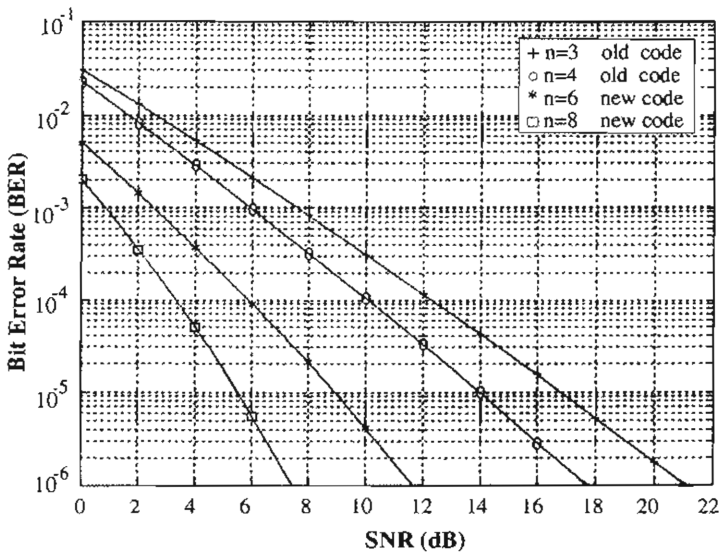

Figure 2: Rate 3/4 space-time block codes from complex orthogonal design (old codes) of n = 3,4 and space-time block codes from constituent intersectant orthogonal design (new codes) of $n = 6,8$ ($m = 1$)

CONCLUSION

In this paper, we introduced the space-time block codes based on constituent intersectant orthogonal designs. The space-time block codes from constituent intersectant orthogonal design have the same low decoding complexity as the space-time block codes from complex orthogonal design. Rate l space-time block codes from constituent intersectant orthogonal design exist for two, three and four transmit antennas while space-time block codes from complex orthogonal design exists only for two transmit antennas. Rate 3/4 space-time block codes from constituent intersectant orthogonal design exist for five, six, seven and eight transmit antennas while space-time block codes from complex orthogonal design exist only for three and four transmit antennas.

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